Definitions of Trigonometric Functions:

If (x, y) is a point on the terminal side of an angle α in standard position and $r=\sqrt{x^2+y^2}$, then

$$\sin\alpha=\frac{y}{r},\qquad \cos\alpha=\frac{x}{r},\qquad \tan\alpha=\frac{y}{x},\qquad \csc\alpha=\frac{r}{y},\qquad \sec\alpha=\frac{r}{x},\qquad \cot\alpha=\frac{x}{y}$$

Reciprocal Functions:

$$\sin x=\frac{1}{\csc x}\qquad \cos x=\frac{1}{\sec x}\qquad \tan x=\frac{1}{\cot x}\qquad \csc x=\frac{1}{\sin x}\qquad \sec x=\frac{1}{\cos x}\qquad \cot x=\frac{1}{\tan x}$$

Tangent and Cotangent in Terms of Sine and Cosine:

$$\tan x=\frac{\sin x}{\cos x}\qquad \cot x=\frac{\cos x}{\sin x}$$

Pythagorean Identities:

$$\sin^2 x+\cos^2 x=1$$
$$\tan^2 x+1=\sec^2 x$$
$$1+\cot^2 x=\csc^2 x$$

Odd/Even Identities:

$$\sin(-x)=-\sin x\qquad \csc(-x)=-\csc x$$
$$\tan(-x)=-\tan x\qquad \cot(-x)=-\cot x$$
$$\cos(-x)=\cos x\qquad \sec(-x)=\sec x$$

Cofunction Identities:

$$\sin\left(\tfrac{\pi}{2}-x\right)=\cos x\qquad \cos\left(\tfrac{\pi}{2}-x\right)=\sin x\qquad \sin(90^\circ-x)=\cos x\qquad \cos(90^\circ-x)=\sin x$$
$$\tan\left(\tfrac{\pi}{2}-x\right)=\cot x\qquad \cot\left(\tfrac{\pi}{2}-x\right)=\tan x\qquad \tan(90^\circ-x)=\cot x\qquad \cot(90^\circ-x)=\tan x$$
$$\sec\left(\tfrac{\pi}{2}-x\right)=\csc x\qquad \csc\left(\tfrac{\pi}{2}-x\right)=\sec x\qquad \sec(90^\circ-x)=\csc x\qquad \csc(90^\circ-x)=\sec x$$

Sum and Difference Identities:

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Double-Angle Identities:

$$\sin(2x)=2\sin x\cos x$$
$$\cos(2x)=\cos^2 x-\sin^2 x$$
$$\cos(2x)=1-2\sin^2 x$$
$$\cos(2x)=2\cos^2 x-1$$
$$\tan(2x)=\frac{2\tan x}{1-\tan^2 x}$$

Half-Angle Identities:

$$\cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}$$
$$\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}$$
$$\tan\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$$

Key Points:

$y = \sin x$

x	0	$\pi/2$	π	$3\pi/2$	2π
y	0	1	0	-1	0

$y = \cos x$

x	0	$\pi/2$	π	$3\pi/2$	2π
y	1	0	-1	0	1

$y = \csc x$

x	0	$\pi/2$	π	$3\pi/2$	2π
y	undef	1	undef	-1	undef

$y = \sec x$

x	0	$\pi/2$	π	$3\pi/2$	2π
y	1	undef	-1	undef	1

$y = \tan x$

x	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$
y	undef	-1	0	1	undef

$y = \cot x$

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	undef	1	0	-1	undef

To Graph $y = a\,f\left[b\left(x-c\right)\right]+d$:

1. Start with the key points of $y = f\left(x\right)$.
2. Multiply the y-coordinates by a and add d.
3. Divide the x-coordinates by b and add c.
4. Plot the new points and connect to form the graph.

Amplitude: $\text{amplitude} = |a|$ $\left(\text{for sin or cos}\right)$

Period:

$$\text{period} = \frac{2\pi}{b} \ \left(\text{for sin, cos, csc, and sec}\right) \text{ or}$$

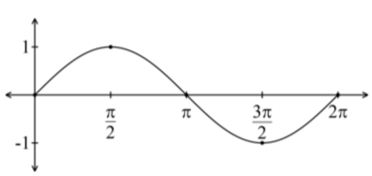
$$\text{period} = \frac{\pi}{b} \ \left(\text{for tan and cot}\right)$$

Frequency: $\text{frequency} = \dfrac{1}{\text{period}}$

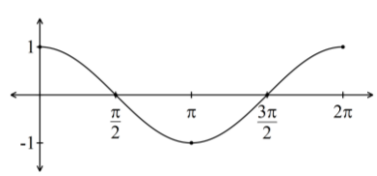
Phase Shift: c (positive if shifted right, negative if shifted left)

Vertical Shift: d (positive if shifted up, negative if shifted down)

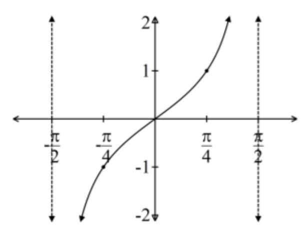
Asymptotes: $x = \text{first non-negative asymptote} + \left(\text{distance between asymptotes}\right) \cdot k$



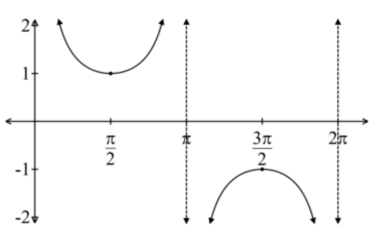
$y = \sin x$



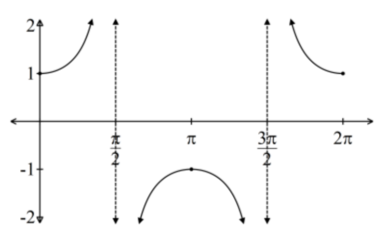
$y = \cos x$



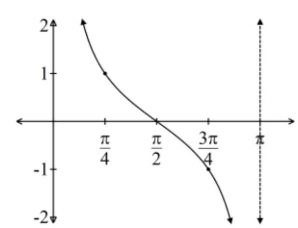
$y = \tan x$



$y = \csc x$



$y = \sec x$



$y = \cot x$

Arc Length:

$$s = \alpha r \ (\alpha \text{ must be in radians}) \ \text{ or } \ s = \frac{\alpha}{360°} \cdot \text{circumference} \ (\alpha \text{ in degrees})$$

Sector Area:

$$A = \frac{\alpha r^2}{2} \ (\alpha \text{ must be in radians}) \ \text{ or } \ A = \frac{\alpha}{360°} \cdot \text{area of circle} \ (\alpha \text{ in degrees})$$

Inverse Functions:

$\sin^{-1} x$ is the angle in $[-90°, 90°]$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is *x*.

$\cos^{-1} x$ is the angle in $[0°, 180°]$ or $[0, \pi]$ whose cosine is *x*.

$\tan^{-1} x$ is the angle in $[-90°, 90°]$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose tangent is *x*.

$\csc^{-1} x$ is the angle in $[-90°, 90°]$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose cosecant is *x*.

$\sec^{-1} x$ is the angle in $[0°, 180°]$ or $[0, \pi]$ whose secant is *x*.

$\cot^{-1} x$ is the angle in $[0°, 180°]$ or $[0, \pi]$ whose cotangent is *x*.

The Law of Sines:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

The Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Area of a Triangle:

$$A = \tfrac{1}{2} bc\sin\alpha$$
$$A = \tfrac{1}{2} ac\sin\beta$$
$$A = \tfrac{1}{2} ab\sin\gamma$$

Heron's Formula:

$$A = \sqrt{S(S-a)(S-b)(S-c)}, \text{ where } S = \frac{a+b+c}{2}$$

Component Form of a Vector with Magnitude r and Direction Angle θ:

$$\langle r\cos\theta, \ r\sin\theta\rangle$$

Magnitude and Direction Angle of a Vector $\langle x, y\rangle$:

$$|\langle x, y\rangle| = r = \sqrt{x^2 + y^2}, \ \sin\theta = \frac{y}{r}, \ \cos\theta = \frac{x}{r}, \ \tan\theta = \frac{y}{x}$$

Dot Product of Two Vectors:

$$\mathbf{A} \cdot \mathbf{B} = \langle a_1, a_2\rangle \cdot \langle b_1, b_2\rangle = a_1 b_1 + a_2 b_2$$

Angle Between Two Vectors:

$$\cos\alpha = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$$

Absolute Value or Modulus of a Complex Number:

$$|a+bi| = \sqrt{a^2+b^2}$$

Trigonometric Form of a Complex Number $z = a + bi$:

$z = r(\cos\theta + i\sin\theta)$, where

$r = \sqrt{a^2+b^2}$, and $\sin\theta = \frac{b}{r}$, $\cos\theta = \frac{a}{r}$, $\tan\theta = \frac{b}{a}$

Product and Quotient of Complex Numbers:

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then

$z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$ and

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$.

De Moivre's Theorem:

If $z = r(\cos\theta + i\sin\theta)$ is a complex number and n is any positive integer, then

$z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]$.

***n*th Roots of a Complex Number:**

For any positive integer n, the complex number $z = r(\cos\theta + i\sin\theta)$

has exactly n distinct nth roots given by:

$r^{1/n}\left[\cos\alpha + i\sin\alpha\right]$, where $\alpha = \frac{\theta + 360^\circ k}{n}$ or $\alpha = \frac{\theta + 2k\pi}{n}$, for $k = 0, 1, 2, \ldots, n-1$.

Polar Coordinates to Rectangular Coordinates:

To convert (r,θ) to rectangular coordinates (x,y), use $x = r\cos\theta$ and $y = r\sin\theta$.

Rectangular Coordinates to Polar Coordinates:

To convert (x,y) to polar coordinates (r,θ), use $r = \sqrt{x^2+y^2}$ and $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$.

Converting Equations Between Rectangular and Polar:

Use $x^2 + y^2 = r^2$, $x = r\cos\theta$, and $y = r\sin\theta$.